

Mass function of stars in the solar neighbourhood

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Summary. Using a new law of the star formation proposed by Rana and Wilkinson (1986a) and some recent data on Luminosity function and scale heights of main sequence disc stars in the solar neighbourhood, the mass function of these stars is investigated and compared with those of Scalo (1986) and Larson (1986). It is found that the classical views such as near constancy of star formation rate, an IMF having a simple power law in mass, no appreciable infall of matter from the halo, and no exotic amount of dark remnants or brown dwarfs, are still able to offer a consistent view of the chemical evolution of the Solar neighbourhood.

Key words: solar neighbourhood – luminosity function – mass function – chemical evolution – star formation rate

1. Introduction

The distribution of stars according to their masses, called the mass function of the stars, was first systematically investigated by Salpeter (1955), who found a slope of -1.35 for stars in the solar neighbourhood, and in the mass range 1 to $10 M_{\odot}$. Subsequently, Miller and Scalo (1979) derived the present day mass function (PDMF in short) $\phi_{ms}(\log m)$ as the number of stars per unit surface area of the disc and per unit logarithmic range of mass, from the observed luminosity function $\phi_{LF}(M_V)$ of the stars in the solar neighbourhood. Here m denotes mass of a star in solar unit and M_V is the absolute visual magnitude of the star. From the PDMF, they finally proposed a form for the initial mass function (IMF in short) of the stars $\xi(\log m)$. The IMF is basically an accumulated frequency distribution of stellar masses and expressed as number of stars born per unit interval of logarithmic mass per unit volume or surface area of the disc. Miller and Scalo derived the possible forms of the IMF in terms of per unit surface area of the disc, assuming various possible forms of evolution of the gross rates of star formation. However, they finally preferred a constant birth rate of about $5 M_{\odot} \text{ pc}^{-2} \text{ Gy}^{-1}$ in the solar neighbourhood and showed that the slope of the IMF was much flatter for low mass stars than suggested by Salpeter and that the slope appeared to be steeper than 1.35 above $10 M_{\odot}$. An approximate representation of the variation of the slope with mass was suggested by them as $-(1 + \log m)$.

Later on, Garmany et al. (1982), Bisiacchi et al. (1983), Vanbeveren (1984) and van Buren (1985) derived somewhat dif-

ferent PDMFs from the one suggested by Miller and Scalo, for stars more massive than about $20 M_{\odot}$. Garmany et al. (1982) studied stars more massive than $30 M_{\odot}$, all lying within 3 kpc from the Sun, and found that the outer half (made of those which lie farther from the centre of the Galaxy than the Sun) is significantly less populated (almost by a factor of 3) than the inner half. From this observation, they concluded that this discrepancy may have arisen due to a spatial variation of the slope of the IMF with the galactocentric distance (R_G).

Güsten and Mezger (1983), and recently Larson (1986) also felt that IMF in the Galaxy is possibly bimodal in its mass distribution – one peak representative of the arm population and the other of the interarm population.

At the lower mass end, some new studies of faint stars, for example, Gilmore and Reid (1983), Eaton et al. (1984), Gilmore et al. (1985), D'Antona and Mazzitelli (1985, 1986), Hawkins (1986), Robin and Crézé (1986) have appeared after the classical work of Miller and Scalo (1979). Some of them have appeared even after Scalo (1986)'s latest review of the subject went to the press. Therefore, making use of these and some other new input data, particularly on the scaleheights, we derive plausible forms of the PDMF and IMFs of the stars in the solar neighbourhood and compare with Scalo (1986)'s results and also Larson (1986)'s. Our final aim has been to use these results for studying the possible consequences for the history of chemical evolution and density of remnants in the solar neighbourhood.

In Sect. 2, we plan to compile various observational and theoretical results required for derivation of the PDMF of stars in the solar neighbourhood, followed by a section on the derivation of the IMF.

2. Derivation of the PDMF of stars in the solar neighbourhood

The derivation of the PDMF involves a number of steps each of which rests on some observational analysis of data. In the following subsections, we shall briefly go through each of them. For details of procedures, one is referred to the works by Miller and Scalo (1979) and Scalo (1986).

2.1. Luminosity function of stars $\phi_{LF}(M_V)$

$\phi_{LF}(M_V)$ is defined to be the number of main sequence stars per unit volume, each star having an absolute visual magnitude in the range from $M_V - 0.5$ to $M_V + 0.5$. We have taken data from

Table 1. Compilation of the luminosity functions $\phi_{\text{LF}}(M_V)$

Absolute visual magnitude M_V	$\log \phi_{\text{LF}}(M_V)$					
	Gilmore and Reid (1983)	Eaton et al. (1984)	Gilmore et al. (1985)	Robin and Crézé (1986)	Our mean	Scalo (1986)
−5	−7.25	−7.07		−6.94	−7.09	−7.43
−4	−6.72	−6.64		−6.34	−6.57	−7.00
−3	−6.10	−6.09		−5.89	−6.03	−6.35
−2	−5.45	−5.53		−5.38	−5.45	−5.73
−1	−4.81	−5.00		−4.93	−4.91	−5.11
0	−4.28	−4.49		−4.47	−4.41	−4.54
1	−3.70	−3.86		−3.77	−3.78	−3.82
2	−3.29	−3.45		−3.36	−3.37	−3.31
3	−2.96	−3.09		−2.90	−2.98	−2.83
4	−2.63	−2.81		−2.66	−2.70	−2.62
5	−2.49	−2.66		−2.51	−2.55	−2.48
6	−2.44	−2.53		−2.46	−2.48	−2.40
7	−2.52	−2.47		−2.48	−2.49	−2.50
8	−2.40	−2.39		−2.41	−2.40	−2.40
9	−2.40	−2.30	−2.65	−2.31	−2.41	−2.24
10	−2.15	−2.19	−2.33	−2.14	−2.20	−2.05
11	−2.00	−2.10	−2.24	−1.98	−2.08	−1.93
12	−1.80		−1.94	−1.83	−1.86	−1.82
13	−1.86		−1.81	−1.76	−1.81	−1.96
14	−2.10		−2.35	(−1.74)	−2.2	−2.16
15	−2.35		−2.63	(−1.74)	−2.5	−2.43
16	−2.55		−2.8	(−1.74)	−2.6	−2.75
17	−2.80		−2.5	(−1.74)	−2.5	
18	< −2.3		−2.1		−2.1	
19	−1.70				−1.7	

Gilmore and Reid (1983), Eaton et al. (1984), Gilmore et al. (1985) and Robin and Crézé (1986), and presented the numbers in Table 1, after duly correcting for the main sequence fractions g_{ms} for the first three sets. The values of g_{ms} are taken from Scalo (1986). The data of Robin and Crézé do not need this correction, because their table was in the pattern of the Hertzsprung-Russell diagram, so that one could straightaway read the desired numbers for the main sequence part only. These numbers also roughly agree with the adopted g_{ms} values in Scalo (1986). For the data given up to $M_V = 19$ by Gilmore and Reid, the statistics are in fact very poor above $M_V = 15$. For a sky coverage of about 18 square degrees and a depth no more than 100 pc, they find at most one or no star at all for $M_V > 16$. However, while dealing with two different parts of the sky with similar coverage of area, Gilmore and Reid (1983) and Gilmore et al (1985) find very similar numbers for all magnitudes $M_V > 8$. Eaton et al. (1984) have their basic data in the $2.2 \mu\text{m}$ (I -band) in the infrared, and independently derived an LF as a function of M_V , ranging from $M_V = -6$ to $M_V = +11$. Robin and Crézé (1986) have carefully reanalysed the classical data of Wielen (1974, 1977), Gliese and Jahreiss (1979), Upgren and Armandroff (1981), Probst (1983), Schmidt (1983), Dahn et al. (1983), and others. The numbers suggested by Robin and Crézé are in good agreement with those mentioned above for $M_V < 14$. For fainter magnitudes, they have made an ad hoc assumption of a flat LF at about $1.8 \cdot 10^{-2}$ stars/pc³/mag; so we have preferred to disregard Robin and Crézé

(1986) for $M_V > +13$. An average of all these LFs are derived for subsequent use. In Fig. 1, plotted are these luminosity functions alongwith the Miller and Scalo's.

It is interesting to note that all the three luminosity functions agree quite well with each other at higher masses, but disagree significantly with Miller and Scalo's at the fainter end. Scalo (1986)'s luminosity function also tends to fall off faster at the brighter visual magnitudes.

2.2. Calibration of mass from the absolute visual magnitudes

Binary stars provide a reliable estimation of both mass and luminosity simultaneously. A few hundred binary stars have so far been studied. We have used Popper (1980)'s exhaustive compilation of all data then available on binary systems and Cester et al. (1983)'s and Grenier et al. (1985)'s more recent additional data. All these data are plotted in Fig. 2, and best fit line is smoothly drawn through these points. This line represents our empirical mass $-M_V$ relation.

The above $m - M_V$ relation departs significantly from the one used by Miller and Scalo (1979) and Scalo (1986), particularly at the low mass end. The middle portion of the empirical relation in Fig. 2 has a number of ripple like variations. When compared with the theoretical calculations of the stellar evolution models by Maeder (1980–1983) for massive stars, the mass-luminosity relation appears to be smoother than the above

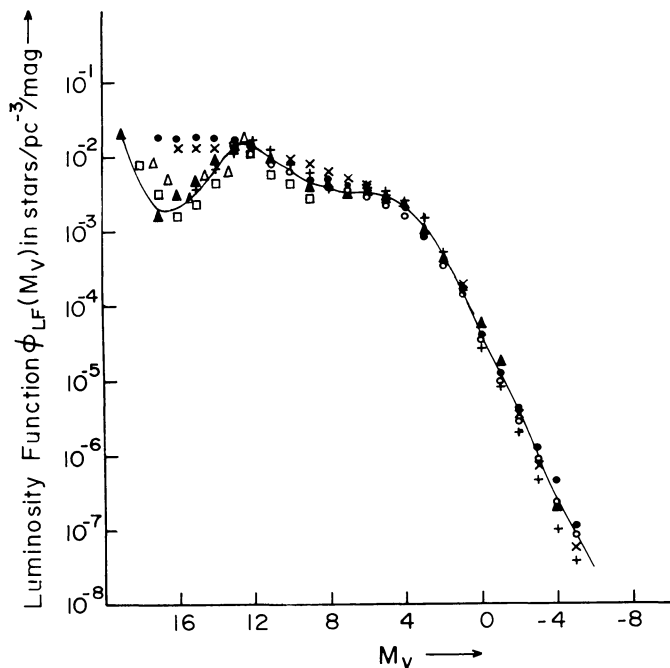


Fig. 1. A compilation of the available luminosity functions of the main sequence disc stars in the solar neighbourhood. The trend of the past few years' data deviates significantly from Miller and Scalzo (1979) at fainter magnitudes and from Scalzo (1986) at brighter magnitudes. One hump and a broad shoulder are evident, though the trend at the faintest magnitudes is not very clear.

- × Miller and Scalzo (1979) ● Robin and Cr    (1986)
- ▲ Gilmore and Reid (1983) + Scalzo (1986)
- Eaton et al (1984) △ Hawkins (1986)
- Gilmore et al (1985)

$m - M_V$ relation. The reason is traced to the non-linear relationship between the bolometric and visual magnitudes, which is shown in Fig. 3. The mass-luminosity relation that has been derived empirically by Cester et al. (1983) fits quite nicely with the theoretical calculations of Maeder (1980). If the slope of the mass-luminosity curve be α , then the slope $-dM_{bol}/d \log m$ should be 2.5 times α . We know that theoretical models suggest $\alpha = 5.5$ for low mass stars ($m \lesssim 1$), and it decreases to 3.5 for intermediate mass stars. For very high mass stars, the index α

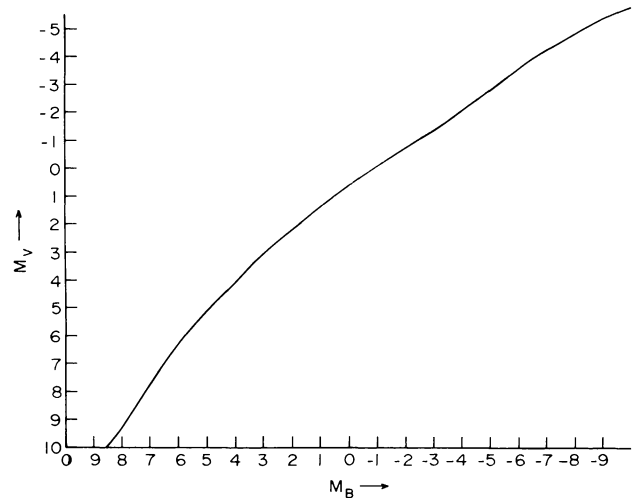


Fig. 3. The empirical relation between visual and bolometric magnitudes of a star. The relation is neither linear nor of steady curvature. Because of such wavy behaviour alone, a smoothly varying mass-luminosity relation may give rise to an undulating $m - M_V$ relation as seen in Fig. 2

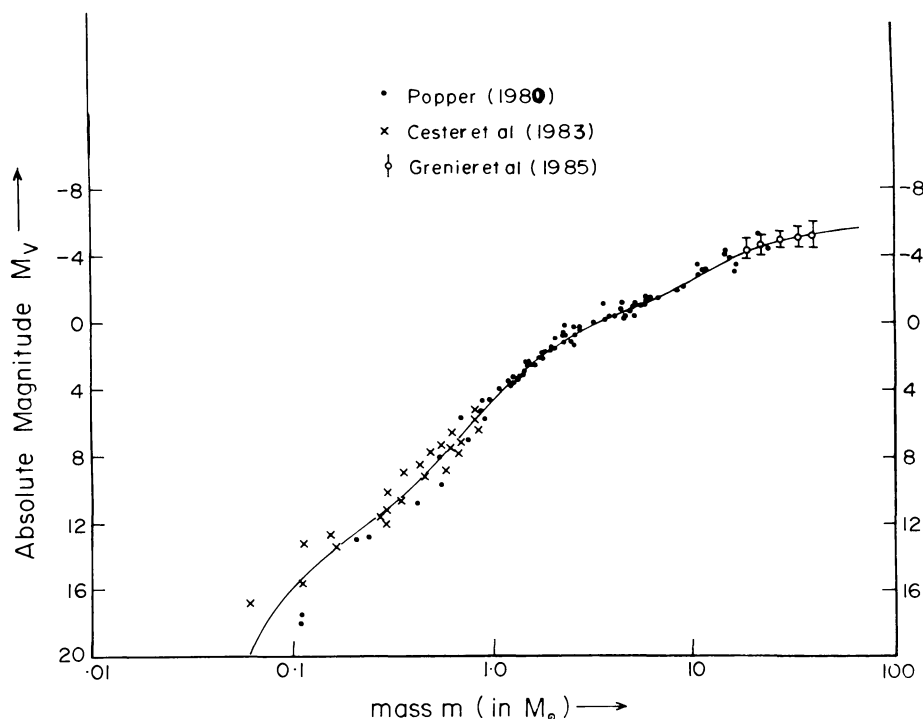


Fig. 2. Empirical calibration of the $m - M_V$ relation for the main sequence stars. Reliable estimates of both mass and luminosity are available only for the nearby binaries. The adopted relation is shown by the solid curve. It significantly deviates from both Miller and Scalzo (1979) and Scalzo (1986), particularly for points below $M_V = 4$

should asymptotically reach unity. So it is not surprising that the magnitude of $-dm_V/d\log m$ for our assumed relation reaches about 14.5 for $m = 0.8-0.9$. In Miller and Scalo, and Scalo's $m - M_V$ relation, the slope never exceeds 10, which means that their implied value of α does not exceed 4 for any value of m .

D'Antona and Mazzitelli (1982, 1983, 1985, 1986) have theoretically investigated the possible forms of the mass-luminosity relation for very low mass stars ($m < 0.5$), and they roughly tally with our relation. Since the visual magnitude falls off very sharply with surface temperature for low mass stars, the slope in terms of M_V keeps on increasing towards the lowest mass end.

From Fig. 2, we have derived m (equivalently, $\log m$ also) and $-dM_V/d\log m$ for the required integral values of M_V , so that with these values a luminosity function can be converted into a mass function.

2.3. Scale heights of stellar distribution in the plane of the disc

The scale heights of the stars are needed if the mass function is to be expressed in terms of the unit area of the disc, rather than per unit volume. Stars of varying luminosity in the main sequence phase are found to have varying amounts of random velocities, which make them oscillate up and down about the mid-Galactic plane with varying spatial amplitudes. Young and bright stars show fairly low dispersion in velocity and are found to be confined within a scale height of about ± 85 pc. These stars form usually inside giant molecular clouds which also have similar scale heights. Because such stars are short lived, they can hardly go far from their place of origin, and also interact hardly with any cloud for enhancement of their random velocities (Wielen, 1977; Vader and de Jong, 1981; Villumsen, 1985). The low mass stars, on the other hand, live much longer and develop much larger random velocities with time, and therefore, spread over a much greater scale height. So the scale heights ($2H$) depends in general upon the mass or the absolute magnitude of the stars. They also reflect the variation in metallicity, which is also regarded as a good indicator of age (Twarog, 1980; Twarog and Wheeler, 1982; Carlberg et al., 1985; Robin and Crézé, 1986).

We have combined all the data on scale heights compiled by Miller and Scalo (1979) and Scalo (1986), and the latest determination by Robin and Crézé (1986). They are shown in Fig. 4. The smooth solid line drawn through these points represents our preferred relation for $H(M_V)$, giving more weight to the latest determination of Robin and Crézé. Apart from the fact that there is a sharp rise in the scale height between $M_V = 2$ and 4, by more than a factor of 2, whether H should soon flatten off at fainter magnitudes or should have a small peak and then flatten off gradually to somewhat lower value of H is anyone's guess. However, since any peak, even a small one, in $H(M_V)$, would be quite difficult to justify from the theoretical standpoint, we prefer a smooth and monotonous line (say, the solid curve) to a humpy one (say, the dashed curve), even though the latter appears to have a better eye-fit. From the IRAS and OH surveys, Habing (1987) has constructed an infrared image of the disc, which is found to have a vertical scale height $H = 0.4-0.55$ kpc, whereas Van der Kruit (1986) suggested $H = 0.6-0.7$ kpc and Bahcall (1986) gave $H = 0.35$ kpc. Our preferred value has been $H = 0.58$ kpc for the faintest magnitudes.

However, the scale heights are better known for massive stars than for the low mass stars, as the completeness of the sampling of faint stars does hardly go beyond 100 pc from the Sun. The

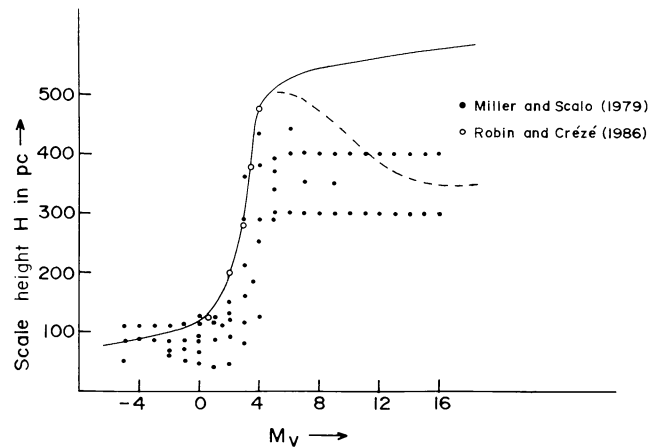


Fig. 4. Scale heights of the disc stars plotted against their absolute visual magnitudes. Considerable scatter is there in the available data, but we have put more weight on Robin and Crézé (1986)'s estimation of the variation of scale heights with the age and metallicity of the stars. The solid curve represents our adopted relation. The same trend of slowly rising H with higher M_V has been maintained throughout despite the fact that a peak in the scale height at about $M_V = +5$ and a slow decline at fainter magnitudes could have been a better fit to the data (dotted curve). The level of present uncertainty in $H(M_V > 5)$ may be guessed from the following statistics: $H = 0.35$ kpc (Bahcall, 1986), $= 0.4-0.55$ kpc (Habing, 1987), $= 0.6-0.7$ kpc (Van der Kruit, 1986), $= 0.5$ (Robin and Crézé, 1986)

bright stars are often traced to distances well beyond 1 kpc, allowing direct observation of the flatness of the disc. So the volume densities of the stellar population in the disc are better estimated than the surface densities, particularly for the low mass stars. But again, it is the whole effective width of the disc that takes part in the final distribution (or evolution) of the disc stars, and therefore, the surface densities are more important than just the volume densities. A knowledge of the scale heights is required for this interconversion.

2.4. The present-day mass function (PDMF)

The PDMF is a product of three quantities, namely, the luminosity function, the magnitude of the slope of $M_V - \log m$ plot and the scale height, and is given by

$$\phi_{ms}(\log m) = \phi_{LF}(M_V) \cdot \frac{dM_V}{d\log m} \cdot 2H$$

These quantities are tabulated in Table 2, and are plotted in Fig. 5. We have also plotted the PDMFs derived by Garmany et al. (1982), Bisiacchi et al. (1983) and Humphreys and McElroy (1984) for stars more massive than $20 M_\odot$, so that the range of the PDMF now covers from about $0.01 M_\odot$ to about $100 M_\odot$. Van Buren (1985)'s work has not been included here partly because he mainly used the same sample as Garmany et al.'s and partly because reduction to PDMF was not straightforward. For comparison, Scalo (1986)'s estimates are also plotted. Then, in Fig. 6, we have plotted the product of mass and the PDMF, that is, $m\phi_{ms}(\log m)$ against $\log m$ in order to get an idea of the differential distribution of total mass that presently resides in the form of main sequence stars. We have smoothly joined our data

Table 2. Present-day mass function $\phi_{\text{ms}}(\log m)$

M_V	$\log \phi_{\text{LF}}$	$\log m$	m	$-\frac{d \log M_V}{d \log m}$	2H (in pc)	$\log \phi_{\text{ms}}$ (log m)	$m \phi_{\text{ms}}(\log m)$
−5	−7.09	1.48	30.5	3.2	170	−4.36	0.0013
−4	−6.59	1.25	17.8	5.3	180	−3.59	0.0046
−3	−6.03	1.08	12.0	6.9	190	−2.91	0.015
−2	−5.45	0.92	8.4	5.6	200	−2.40	0.033
−1	−4.91	0.73	5.4	4.9	215	−1.89	0.069
0	−4.41	0.54	3.45	5.6	240	−1.28	0.18
1	−3.78	0.39	2.45	7.8	290	−0.43	0.91
2	−3.37	0.26	1.82	8.5	390	0.15	2.6
3	−2.98	0.16	1.43	10.0	570	0.78	8.7
4	−2.70	0.06	1.14	11.0	940	1.32	24.0
5	−2.55	−0.02	0.95	14.5	1030	1.62	39.8
6	−2.48	−0.09	0.81	14.5	1060	1.71	41.7
7	−2.49	−0.16	0.69	13.2	1070	1.66	31.6
8	−2.40	−0.24	0.57	12.0	1080	1.71	29.5
9	−2.41	−0.33	0.47	11.4	1090	1.75	26.3
10	−2.20	−0.42	0.38	11.1	1100	1.93	32.4
11	−2.08	−0.51	0.31	10.1	1115	2.02	32.4
12	−1.86	−0.62	0.24	8.6	1125	2.17	35.5
13	−1.81	−0.73	0.19	9.5	1135	2.22	30.9
14	−2.2	−0.83	0.15	10.4	1140	1.97	13.8
15	−2.5	−0.92	0.12	11.4	1150	1.77	7.1
16	−2.6	−1.00	0.10	13.5	1155	1.74	5.5
17	−2.5	−1.08	0.084	15.3	1160	1.95	7.5
18	−2.1(?)	−1.13	0.074	17(?)	1160(?)	2.2(?)	11.7
19	−1.7(?)	−1.18	0.066	22(?)	1160(?)	2.7(?)	34(?)
		−1.4				0.8(?)	0.3(?)
		−2.0				1.1(?)	0.1(?)

points in Fig. 6 for the purpose of integrating the area between any two mass limits.

The following comments may be made from Figs. 5 and 6.

(i) In Fig. 5, the data for massive stars given by Garmany et al., Bisiacchi et al. and Humphreys and McElroy together with ours are roughly seen to follow the same trend, indicating that independent estimates are overlapping. This is not so with Scalo's data, the departure being generally larger than his estimated error limits.

(ii) Scalo (1986)'s data with his upper limit of error bars are found to be systematically falling short of our estimates. This of course is partly due to larger values of H and the $m - M_V$ slope. This effect was however already noticeable in the plot of the luminosity functions. The reason why Scalo's PDMF falls below others is that Scalo's adopted masses of stars are smaller for a given luminosity. He has corrected for possible brightening due to non-local convection.

(iii) When integrated between $m = 0.08$ and $m = 100$, our PDMF in Fig. 8 accounts for about $\Sigma_* \simeq 32 M_\odot \text{pc}^{-2}$ in the form of visible main sequence stars, compared to Scalo's about $22 M_\odot \text{pc}^{-2}$ for his tabulated values. The error bars in Scalo's analysis, which look so innocent in Fig. 5, give an idea of the real uncertainties present in PDMF determinations, particularly when plotted in any linear form (as in Fig. 6), suggesting the total surface density of visible stars in the solar neighbourhood to be anything between 13 and $39 M_\odot \text{pc}^{-2}$. This may be

compared with the total surface density of matter (Σ_T) from dynamical arguments suggesting about $75 M_\odot \text{pc}^{-2}$, although the real scatter may allow any figure in the range 60 to $150 M_\odot \text{pc}^{-2}$ (Bahcall et al., 1983). According to our PDMF, the visible stars can account for about 50 percent if $\Sigma_T = 67 M_\odot \text{pc}^{-2}$, or about 43% if $\Sigma_T = 75 M_\odot \text{pc}^{-2}$.

We also studied the effect of changing our preferred $H(M_V)$ relation, from the solid curve shown in Fig. 4 to the dotted one, for example. If the flat value of $H(M_V)$ at the lowest magnitude approaches to $H_3 \text{ kpc}$, the integrated PDMF for our LF would roughly correspond to

$$\Sigma_* \simeq 11 + 35H_3 M_\odot \text{pc}^{-2}.$$

Therefore, the dependence of Σ_* on H_3 is very strong, and hence their mutual uncertainties. This relation satisfies Scalo's estimate of $22 M_\odot \text{pc}^{-2}$ for his $H_3 = 0.325$

(iv) The tabulated values, that is, the points themselves in Fig. 5 suggest two peaks in the mass distribution of the PDMF. The peaks are not in identical positions in the two sets. Our PDMF suggests the peaks to be at $m \simeq 0.2$ and 0.9 , whereas Scalo's PDMF indicates at $m \approx 0.5$ and 0.9 respectively. However, in view of the large error bars, the dip at about $m \approx 0.6 - 0.7$, known as the Wielen dip, may not be real at all, in which case a smooth PDMF with a single peak may account for about $35 M_\odot \text{pc}^{-2}$ in the form of visible stars. Some people including

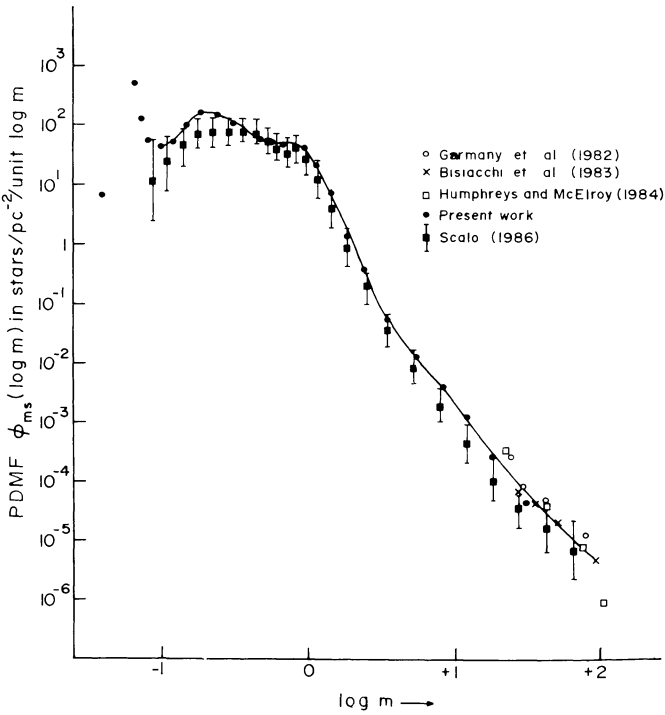


Fig. 5. The present day mass function (PDMF) of the stars in the solar neighbourhood. The present work is compared with that of Scalzo (1986). The latter seems to have systematically lower values at practically all $m > 0.6$, but the departure at lower masses is primarily due to the differences in scaleheights. The PDMF is quite uncertain below $m \approx 0.08$ and the dip at about $m \approx 0.6$, called the Wielen dip, is apparent

Bahcall (1986) have expressed their reservations against such a possibility.

(v) Whether the PDMF again rises towards the lowest mass end is not very clear. The fact that one finds approximately one star in about 18 square degrees of the sky over each magnitude interval from $M_V = +16$ onwards, is an indication of a constant PDMF, if not a slowly rising one. We have considered three possible cases giving weight to either Robin and Cr  z  's assumption of constant LF above $M_V = 13$, or Gilmore et al's LF implying a dip in LF at about $M_V = 17$ and indicating a possible rise in LF at fainter magnitudes, or Scalzo's suggestion of fast decline in LF continuing across the threshold of hydrogen burning.

A flat LF as suggested by Robin and Cr  z  , if extended up to say $m = 0.004$, (the dot-dashed line in Fig. 6) would correspond to a net amount in the form of Jupiter-like brown or black dwarfs to be about $25 M_\odot \text{ pc}^{-2}$. As for the second case, Gilmore and Reid (1983)'s LF with one star per 18 sq. deg. at $M_V = 19$ may provide any amount of dark matter depending on the steepness of the rise in the LF from $M_V = 18$ onwards. On the other hand, Gilmore et al. (1985)'s LF with no star at $M_V = 19$ and in the same solid angle may suggest very little dark matter in the form of brown dwarfs. But again, recent infrared studies (Hawkins, 1986) show a similarly rising LF as in Gilmore and Reid's for $M_V > 17$. So we find at least two independent observations suggesting that there might be an upward trend in the PDMF between $m = 0.1$ and $m = 0.06$ (as shown by the solid curve in Fig. 8). This obviously would go against Larson (1985)'s model of cloud fragmentation which derives the characteristic

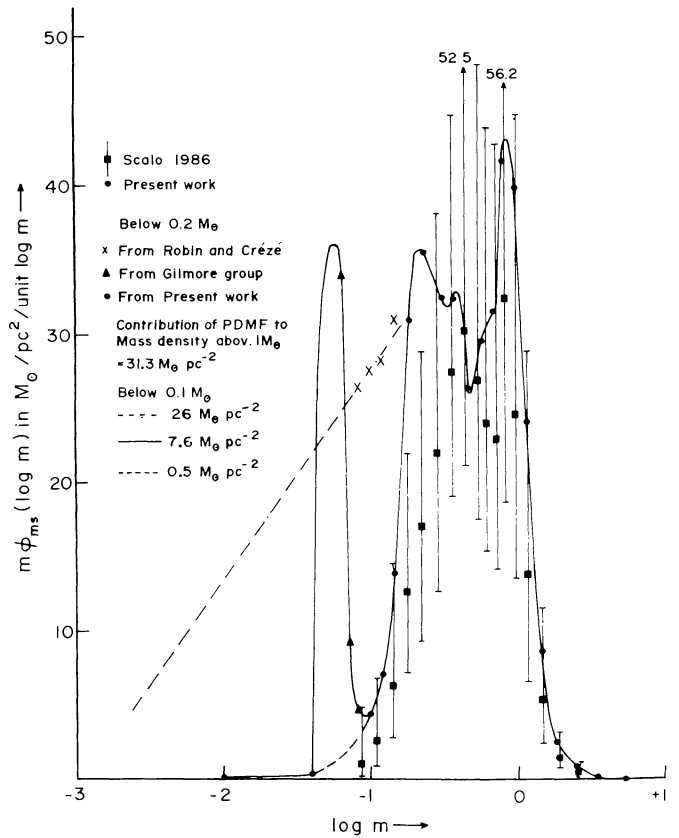


Fig. 6. The distribution of mass in the PDMFs. Since this is a linear-log plot, any small logarithmic error in the ordinate will appear to be inflated around the peak. The reality of the Wielen dip in the mass distribution becomes questionable. Our PDMF accounts for about $31.3 M_\odot \text{ pc}^{-2}$ for $0.1 < m < 100$, and below $m = 0.1$, the contribution to the total surface density of matter is highly uncertain. The filled circles are joined by smoothly varying lines just to calculate the area under the curve and to point out the danger of taking the tabulated values very seriously.

stellar mass to be $\sim 0.3 M_\odot$. But such a rise in PDMF may not continue far below $m = 0.06$, because Boss (1986) finds that the minimum protostellar mass based on Silk's analytical model of hierarchical opacity-limited fragmentation of clouds of Population I composition is about $0.02 M_\odot$. Again, one may notice that we have added in Table 3 two more rows at the bottom, extrapolated from the scanty knowledge of the frequency distribution of the dark and failed-star companions of some very nearby, known binaries. Within about 10 to 15 pc from the Sun, there are about 7 unseen companions (spectroscopically determined) which have mass in the range of 0.005 – $0.022 M_\odot$, and 7 more in the mass range 0.03 – $0.06 M_\odot$ (van de Kamp, 1986). Of course, they are dark companions of some very faint main sequence stars, and not just single objects, but their orbital eccentricities are very high, possibly indicating that they are captured rather than fragmented from the primaries. In any case, it does not say how many single objects could be there or what their possible vertical scale heights should be. However, when we plotted those two points in Fig. 6, the overall trend seemed to suggest a narrow peak just below the threshold of nuclear burning allowing no less than about $7.5 M_\odot \text{ pc}^{-2}$ below $m = 0.1$ or about 1 brown dwarf of mass $\sim 0.05 M_\odot$ per 8 pc^3 . Nevertheless, the fact remains that

Table 3. Initial mass functions $\xi(\log m)$ for different cases

$\log m$	$\log \phi_{\text{ms}}(\log m)$	$\log T_{\text{ms}}$	IMF for const. SFR and $T_d = 10$ Gy (case A)		IMF for constant SFR and $T_d = 15$ Gy (case B)		IMF for SFR as in Fig. 10 and $T_d = 15$ Gy (case C)	
			$\log \xi(\log m)$	$m\xi(\log m)$	$\log \xi(\log m)$	$m\xi(\log m)$	$\log \xi(\log m)$	$m\xi(\log m)$
2.00	−6.06	6.49	−2.55	0.3	−2.37	0.4	−2.29	0.5
1.95	−5.33	6.51	−1.84	1.3	−1.66	2.0	−1.58	2.3
1.80	−5.00	6.56	−1.56	1.7	−1.38	2.6	−1.30	3.2
1.60	−4.50	6.76	−0.96	4.4	−0.78	6.6	−0.70	7.9
1.40	−3.95	6.88	−0.73	4.7	−0.55	7.1	−0.47	8.5
1.25	−3.59	7.02	−0.61	4.4	−0.43	6.6	−0.35	7.9
1.08	−2.91	7.22	−0.13	8.9	0.05	13.5	0.13	16.2
0.92	−2.40	7.44	0.16	12.0	0.34	18.2	0.42	21.9
0.73	−1.89	7.80	0.31	11.0	0.49	16.6	0.57	20.0
0.54	−1.28	8.24	0.48	10.5	0.66	15.8	0.74	19.1
0.39	−0.43	8.60	0.97	22.9	1.15	34.7	1.23	41.7
0.26	0.15	8.98	1.17	26.9	1.35	40.7	1.43	49.0
0.16	0.78	9.31	1.47	42.7	1.65	64.6	1.71	74.1
0.06	1.32	9.66	1.66	52.5	1.83	77.6	1.82	75.9
−0.02	1.62	9.96	1.66	43.7	1.84	66.1	1.81	61.7
−0.09	1.71	10.25	1.71	41.7	1.71	41.7	1.71	41.7
−0.16	1.66		1.66	31.6				
−0.24	1.71		1.71	29.5				
−0.33	1.75		1.75	26.3				
−0.42	1.93		1.93	32.4	Same as case A		Same as case A	
−0.51	2.02		2.02	32.4				
−0.62	2.17		2.17	35.5				
−0.73	2.22		2.22	30.9				
−0.83	1.97		1.97	13.8				
−0.92	1.77		1.77	7.1				
−1.00	1.74		1.74	5.5				

any suggestion for the PDMF at fainter magnitudes ($M_V > 14$) is highly speculative, not only because LF and $H(M_V)$ are highly uncertain, but also because of highly uncertain behaviour of the $m - M_V$ relation as pointed out by D'Antona and Mazzitelli (1982–86).

3. The initial mass function (IMF)

The IMF $\xi(\log m)$ corresponds to the mass spectrum of the accumulated number of stars per unit area of the disc, that are born so far, whereas the PDMF $\phi_{\text{ms}}(\log m)$ accounts for those stars which are still in their main sequence phase of evolution. So the knowledge of the main sequence life time T_{ms} and the birth rate of the stars as a function of time and mass $C(m, t)$ is essential for deriving an IMF from a given PDMF.

3.1. Mathematical relation between IMF and PDMF

This relation has been discussed in detail in Miller and Scalo (1979). Since our formulation is slightly different, we would like to briefly discuss it below. From the definition of IMF, it is obvious that

$$\xi(\log m) d \log m = \int_0^{T_d} C(m, t) dt dm \equiv \ln 10 \int_0^{T_d} \psi(m, t) d \log m,$$

where T_d is the age of the disc and $\psi(m, t) = mC(m, t)$ is the rate at which mass of gas goes into new stars at time t , per unit mass of the stellar spectrum and per unit area of the disc. Therefore,

$$\xi(\log m) = \ln 10 \int_0^{T_d} \psi(m, t) dt \quad (1)$$

The main sequence life time T_{ms} of a star is a function of m . Let m_d be the maximum mass of a star which can live as long as T_d . For stars of $m \leq m_d$, IMF = PDMF, that is,

$$\xi(\log m) = \phi_{\text{ms}}(\log m), \quad (2)$$

but for stars $m > m_d$,

$$\phi_{\text{ms}}(\log m) = \ln 10 \int_{T_d - T_{\text{ms}}(m)}^{T_d} \psi(m, t) dt, \quad (3)$$

so that the ratio of $\phi_{\text{ms}}(\log m)$ and $\xi(\log m)$ for all $m > m_d$,

$$F(m) = \frac{\phi_{\text{ms}}(\log m)}{\xi(\log m)} = \frac{\int_{T_d - T_{\text{ms}}(m)}^{T_d} \psi(m, t) dt}{\int_0^{T_d} \psi(m, t) dt}. \quad (4)$$

However, if there is any appreciable mass loss of the massive stars in their main sequence phase, the argument m in $\psi(m, t)$ on the right hand side of (3) should be replaced by

$$m + \int_t^{T_d} \dot{m}(t') dt',$$

where $\dot{m}(t)$ is the mass loss rate of a star which is scheduled to become a star of mass m at $t = T_d$.

Therefore, in order to derive IMF from PDMF, one has to compute the ratio of the two integrals as given in (4), requiring an *a priori* knowledge of $\psi(m, t)$ and $T_{ms}(m)$. However, instead of deriving an IMF from a PDMF, one may also use the PDMF in order to get an idea of $\psi(m, t)$. Miller and Scalo had tried both and concluded that the continuity of the IMF across $m = m_d$ required a constant or very slowly evolving $\psi(m, t)$, the total range of variation to be within a factor of two to five.

However, (4) can be simplified, if one assumes that $\psi(m, t)$ is separable in variables, namely as a product of say, $f_m(m)$ and $\psi_s(t)$, with the result that

$$F(m) = 1, \quad \text{for } m \leq m_d$$

$$F(m) = \frac{\int_{T_d - T_{ms}(m)}^{T_d} \psi_s(t) dt}{\int_0^{T_d} \psi_s(t) dt}, \quad \text{for } m > m_d, \quad (5)$$

where $\psi_s(t)$ is the gross rate of star formation in say $M_\odot \text{ pc}^{-2} \text{ Gy}^{-1}$, in the solar neighbourhood. Such a separation of variables is possible only if $f_m(m)$, that is, the spectral shape of $\psi(m, t)$, is absolutely time independent. Expression (5) can further be simplified, if the star formation rate $\psi_s(t)$ is regarded as a constant, leading to

$$F(m) = 1, \quad \text{for } m \leq m_d,$$

$$F(m) = T_{ms}(m)/T_d, \quad \text{for } m > m_d. \quad (6)$$

3.2. Main-sequence lifetime of stars $T_{ms}(m)$

The main sequence lifetime T_{ms} is defined to be the life of the star devoted to hydrogen burning at its core, and is computed on the basis of theoretical calculations of stellar evolution. For stars less massive than $5 M_\odot$, we have taken data from Tayler (1981) and Tinsley (1980). For stars having $m > 5 M_\odot$, we have compiled from a series of works done by Maeder during 1981–83. The above data are plotted in Fig. 7 and a smooth line is drawn through these points. This line represents our $T_{ms}(m)$ relation, which has been used for computing the values of T_{ms} quoted in the third column of Table 3, and may be approximated by the following quintics.

$$\log T_{ms}(m) = 9.88 - 3.840(\log m) + 1.344(\log m)^2 + 0.107(\log m)^3$$

$$- 0.202(\log m)^4 + 0.040(\log m)^5,$$

$$\text{for } 0.1 < m < 300$$

$$= 6.39, \quad \text{for } m \geq 300.$$

In the above relation T_{ms} is obtained in y. The inverse relation would be

$$\log m = 0.254 - 0.3400(\log T_q) + 0.0471(\log T_q)^2$$

$$+ 0.0102(\log T_q)^3 + 0.0039(\log T_q)^4 - 0.0052(\log T_q)^5,$$

$$\text{for } -2.3 < \log T_q < 3.0$$

Here T_q is T_{ms} expressed in Gyr. The forward relation given above differs significantly from Bahcall and Piran (1983)'s for $m > 10$ and also from Scalo (1986), the latter shown by the dotted line in Fig. 7. Various estimates of $T_{ms}(m)$ have been excellently reviewed by Scalo (1986) and one is referred to the same for a detailed discussion of the possible scatters in the relation, particularly due to convective overshooting (whose validity has been

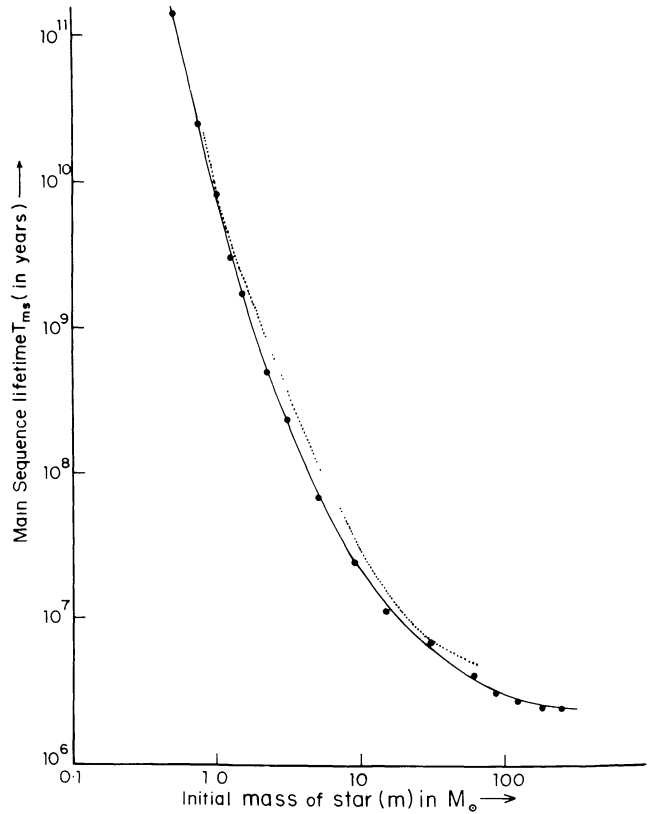


Fig. 7. Mass-lifetime relation for the main sequence stars in the disc. The solid line represents our adopted relation for the population I type of stars. Because of the uncertainty in the mass loss rates of the stars and in the opacity rules, the numbers are subject to likely revisions, but as such the typical error in the above plot may be taken as ± 0.04 dex. The dotted line is due to Scalo (1986)

seriously questioned by Baker and Kuhfub, 1986) in massive stars, the effect of which was to lengthen the main sequence life time.

Apart from the initial mass, T_{ms} also depends on the initial chemical composition. The above numbers refer to the Population I type of stars with near solar metallicity. The Population II type of stars with much lower metallicities can have significantly higher main sequence life times (Stein and Cameron, 1966). Since the disc stars are all Population I type stars and they have metallicity variations in a narrow range, the dependence of T_{ms} on t has been neglected.

3.3. Age of the disc T_d

Stars in the solar neighbourhood are usually classified in several populations according to metallicity content and magnitude of velocity dispersion. Even though these two criteria seem to be mutually independent, they are in fact, not. Both the metallicity and the velocity dispersion of stars from generation to generation increase (with time), as noted by Wielen (1977), Twarog (1980), Vaughan and Preston (1980) and Duncan (1981). The age of the Population II type or extreme metal-poor objects is estimated to be in the range of 11–20 Gyr (Tayler, 1984). The age of the disc of our Galaxy T_d is thought to be between 9 and 15 Gyr (Miller and Scalo, 1979). In recent years many people (for

example, Carlberg et al., 1985; Lacey and Fall, 1985; Larson, 1986; Rana and Wilkinson, 1986a) have taken T_d to be 15 Gyr. However, if the stellar populations are divided distinctly into halo, spheroid, intermediate Population II, and Population I (or disc stars), the age of the disc would possibly be more reasonable to take 10 Gyr, and correspondingly, the initial metallicity of the disc might have been about 0.50 dex below the present metallicity. We shall consider below the two extreme cases, namely $T_d = 10$ and 15 Gyr.

3.4. Variation of the local star formation rate with time $\psi_s(t)$

The actual process of star formation is so far little understood. It possibly depends on metallicity, pressure and temperature of the interstellar gas, magnetic field, and the details of the trigger mechanisms. Since any trigger mechanism is likely to operate in a discontinuous fashion, the star formation may take place only in bursts, leaving $\psi_s(t)$ lost in details (see for example Scalo and Struck-Marcell, 1986). Over a long enough period, should an averaged $\psi_s(t)$ be well behaved, one can work with such a $\psi_s(t)$. Based on some empirical studies, Rana and Wilkinson (1986a) have suggested an empirical formula for $\psi_s(t)$ to be applicable anywhere in the Galaxy. Dopita (1985) has suggested another, based on some theoretical arguments, but it is found to be difficult to express it as explicit function of time. Apart from this, we have some reservations against Dopita's form, which neglects any contributions to SFR by the molecular gas, whereas, we believe there is hardly any correlation between the SFR and the atomic gas (Fig. 8). This point has more elaborately been discussed in Rana and Wilkinson (1986a,b).

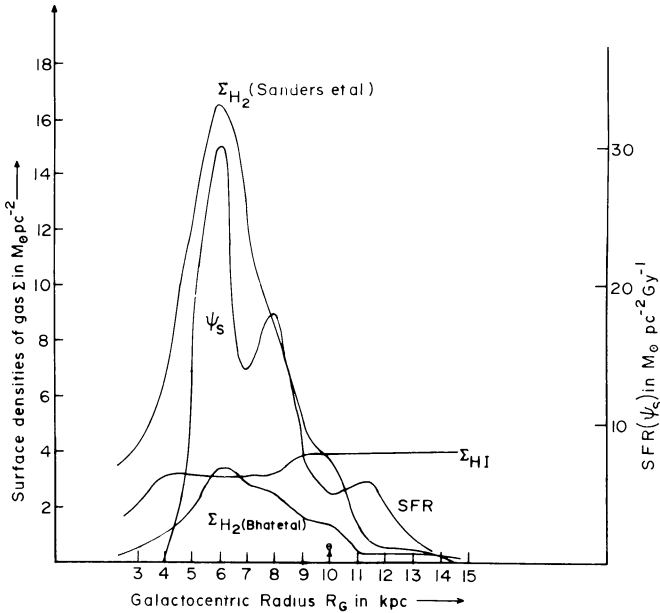


Fig. 8. Surface density distributions of H I and H₂ (that is, $\Sigma_{H I}$ and Σ_{H_2} respectively), and the star formation rates (ψ_s) as functions of the galactocentric radius (R_G) are shown. The two estimates of Σ_{H_2} do not agree with each other, although their variation with R_G roughly conforms with the variation of ψ_s . But no such correlation is apparent between the distributions of $\Sigma_{H I}$ and ψ_s , indicating that most probably H₂, and not H I, has a dominant role in star formation in the disc

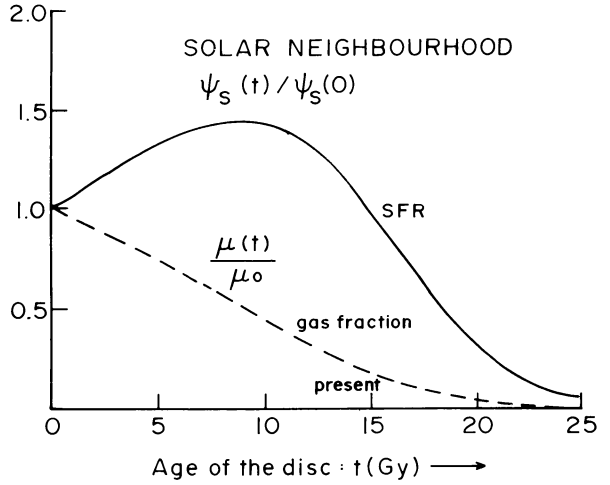


Fig. 9. The time variation of the star formation rate in the solar neighbourhood according to the law of star formation suggested by Rana and Wilkinson (1986a,c)

Therefore, we shall use our $\psi_s(t)$, given in Rana and Wilkinson (1987c) and shown here in Fig. 9, as an example of likely variation of ψ_s with time. Depending on the initial conditions of the disc, for example, the initial metallicity z_0 at the time of the formation of the disc, the then gas fraction μ_0 and the total surface density of matter in the region under consideration, an exact form of our $\psi_s(t)$ may differ by a factor of 2–3, allowing the SFR to vary in different ways, including monotonously increasing to monotonously decreasing with time. Kennicutt (1983) has studied the possible variation of $\psi_s(t)$ in a sample of 77 Sbc–Sd galaxies to find that it stays roughly constant. We also have extended our studies to include several nearby spirals (Rana and Wilkinson, in preparation), which seem to support the generality of our law of star formation. It simply suggests that SFR is proportional to the local surface density of molecular hydrogen raised to power k ($= 1.4 \pm 0.2$), and the molecular fraction of the gas by mass is found to be correlated with the average metallicity of the region, again fitted with a power law giving the index $b = 1.3 \pm 0.3$ for the Galaxy. Apparently the relationship is (Rana and Wilkinson 1986a)

$$\psi_s \propto \mu^k Z^{kb}, \quad (7)$$

where μ is the gas fraction and Z is the metallicity of the region.

3.5. Initial mass function (IMF)

With the knowledge of the PDMF, age of the disc, main sequence life-time of stars and the SFR, the IMF has been derived below for three different cases, which are tabulated in Table 3. It is to be noted that the PDMF and IMF are the same for $m \leq m_d$, where $T_{ms}(m_d) = T_d$ = the age of the disc. The three cases are as follows:

- (A) A constant SFR and $T_d = 10$ Gyr,
- (B) A constant SFR and $T_d = 15$ Gyr,
- (C) SFR as given by Fig. 9 and $T_d = 15$ Gyr.

Now $\log[\xi(\log m)]$ and $m\xi(\log m)$ are plotted against $\log m$ in

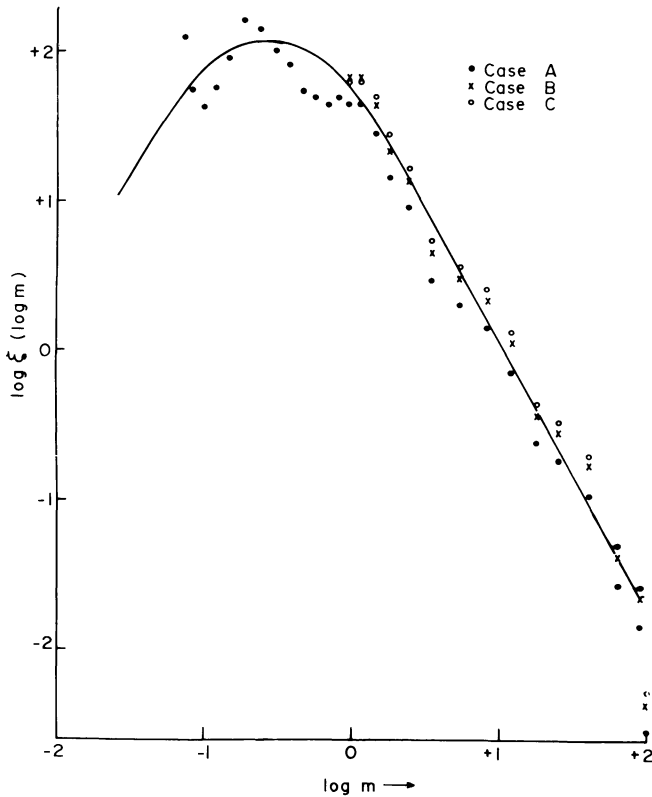


Fig. 10. The initial mass function (IMF) of main sequence stars of the disc population around the Sun. Three forms are suggested in order to give an idea of the basic uncertainties involved in the steps linking a PDMF to an IMF. Given the uncertainties, as above, a single power law seems to fit although, from $m \approx 1.5$ to about $m \approx 100$. On the low mass side, no such simple power law seems to fit the data, although a quadratic power law has been tried (the solid curve)

Figs. 10 and 11 respectively, in the same way as we did for PDMF in Figs. 5 and 6.

4. Discussion

4.1. A general form of the IMF

The differences among these three specific cases (A)–(C) in Sect. 3.5 are appreciable for the entire region $m > 1$. One may regard these differences as a measure of the minimum uncertainty in our present day knowledge of SFR and T_d and hence of $\xi(\log m)$. Further, in view of the uncertainties of the PDMF as estimated by Scalo and shown in Figs. 5 and 6 for $m < 1$, one may possibly ignore the fine scale structures in the IMF and prefer to draw a smooth line although. If we do so, it is then possible to draw a straight line over the entire range $0.2 < \log m < 1.95$, suggesting a constant slope of -1.80 , and for the points below $\log m = 0.2$, a quadratic may be fitted so that the general form of the IMF may be analytically represented by

$$\begin{aligned} \xi(\log m) &= 57m^{-1.20-1.12 \log m-0.06(\log m)^2}, & \text{for } \log m < 0.2 \\ &= 70m^{-1.80}, & \text{for } \log m \geq 0.2 \end{aligned} \quad (8)$$

$\xi(\log m)$ is in unit of number of stars per pc^2 per unit interval of $\log m$. To compare (8) with Scalo (1986), we may note that he

concludes the slope for the field star IMF to be about $-1.70^{+0.4}_{-0.7}$ for $2 \lesssim m \lesssim 10$ and that it is extremely uncertain for $m > 10$. However, for cluster members, he finds the slope to be -1.7 ± 0.2 for $1.5 \lesssim m \lesssim 10$.

Equation (8) may be compared with other recent determinations of the slope of IMF over the high mass range. For example, van Buren (1983) has suggested

$$\xi(\log m) \approx 194m^{-1.0} \text{pc}^{-2} (\log m)^{-1}, \quad \text{for } m > 3.$$

Vanbeveren (1984) has given

$$\xi(\log m) \approx 17m^{-1.30 \pm 0.10} \text{pc}^{-2} (\log m)^{-1}, \quad \text{for } m > 20.$$

Finally, Scalo's (1986) IMF for $T_d = 15$ Gyr can be shown to have a good fit for

$$\xi(\log m) \approx 28m^{-2.12} \text{pc}^{-2} (\log m)^{-1}, \quad \text{for } 1.5 \lesssim m \lesssim 70.$$

4.2. A sign of multimodality?

In favour of small structures that are seen in the form of several humps in Figs. 10 and 11 for all the three cases, one may argue that the IMF is likely to be multimodal with a number of humps (as more evident in Fig. 11). The individual humps seem to occur at reasonably constant intervals of $\log m$, say $\Delta \log m \approx 0.75$ between two successive humps, with an amplitude in $\log \xi$ to be about 0.25. An approximate analytical representation of such a periodic (?) deviation required to be superimposed on the general form such as (8), may be given by

$$\Delta \log \xi(\log m) = 0.25 \cos(8.4 \log m - 1.0). \quad (9)$$

This periodic variation seems to have a few advantages:

First of all, it can explain why the individual determinations of the slopes of $\log \xi(\log m)$ over narrow ranges of mass appear to vary considerably widely, say from -1.3 to -2.4 , depending on the interval of mass one chooses. These slopes often lead to misleading results when unduly extended to either side of the appropriate ranges.

Secondly, on integration over the entire mass range, the value of the integral will not differ by more than about 15%, whether or not (9) is superimposed on (8). The general form (8) would be good enough for any rough calculations involving IMF. The subtle features such as the Wielen dip at $m \approx 0.6$ can be easily reproduced by superposing (9) on (8).

A third point is that unlike Larson's bimodal IMF which suggested a strongly decreasing $\psi_s(t)$ with an unchanging form of $f_m(m)$, or Scalo's time-dependent exponential cut-off in the explicit form of $f_m(m)$ but with roughly constant $\psi_s(t)$, the above proposition of multimodality, if reasonable at all, may still be consistent with the old idea of Miller and Scalo (1979), namely near-constancy of $\psi_s(t)$ and universality and ubiquity of $f_m(m)$. In order to justify such a case, one may argue in the following way.

In Larson's model, the bimodality around $m \gtrsim m_d$ is an artefact of strongly decreasing $\psi_s(t)$; but surely multimodality cannot be generated in the same way. However, Güsten and Mezger (1983) have also advocated bimodal IMF for the Galaxy, one mode for the arm region and the other mode for the interarm region. For a multimodal case, it may be that the star forming cloud undergoes several mini bursts of star formation, each burst giving birth to stars of some specific mass range. Star bursts are one of the current topics that are least understood, nevertheless

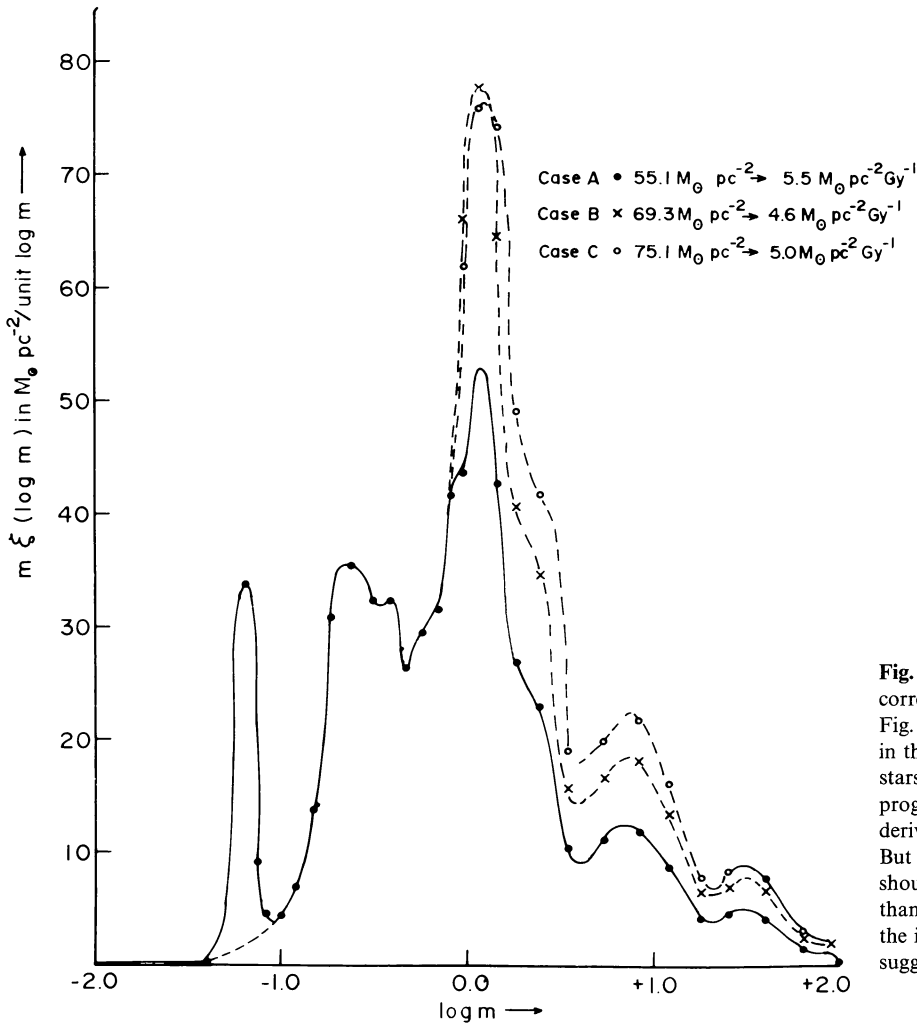


Fig. 11. The mass distribution in the IMFs corresponding to the ones suggested in Fig. 10. A number of humps are apparent in the tabulated values. Since high mass stars live too short compared to their progenitor GMCs, SFR and, therefore, the derived IMF can show some irregularities. But low mass stars that outlive the disc should have led to a far smoother IMF than the one seen in the Figure. May be, the irregularities in the IMF are genuine, suggesting some kind of multimodality.

one may be referred to a speculative physical model of star bursts proposed by Scalo and Struck-Marcell (1986). However, it is known that the theoretical lifetime of dynamically evolving GMCs is only about 10 to 20 Myr, whereas the infrared studies suggest the efficiency of star formation to be hardly 20% (Rengarajan, 1984). All it means is that the cloud has to undergo several cycles of star formation before all the molecular gas it had initially can be used up. In fact, lulled star formation with the peak of $f_m(m)$ shifting towards lower mass end has been seen in the Ophiuchus complex (Turner, 1984). Such a scheme can be an alternative to Larson's bimodality. If extended, it might explain multimodality.

4.3. Star formation rate from the IMF

Taking the individual IMFs for the cases (A)–(C), one can easily compute the total area under each of the curves in Fig. 11, which gives 55.1, 69.3 and $75.1 M_{\odot} \text{ pc}^{-2}$ respectively (for $m > 0.1$), and the corresponding SFR, $\psi_s(t = T_d) = 5.5$, 4.6 and $5.0 M_{\odot} \text{ pc}^{-2} \text{ Gyr}^{-1}$. Even though the total amount varied greatly depending on the choices of mainly T_d , a mildly varying or constant $\psi_s(t)$ would suggest the present day SFR in the solar neighbourhood to be about $5.0 \pm 0.5 M_{\odot} \text{ pc}^{-2} \text{ Gyr}^{-1}$. This again is consistent with Smith et al. (1978), Miller and Scalo, Twarog (1980) and

Scalo (1986). It also tallies with the estimates of SFR from the infrared studies (for example, Iyenger et al., 1985), provided the surface density of molecular gas is taken to be the one given by Bhat et al. (1985).

4.4. Remnants of the massive stars

If a star of mass m leaves a dark remnant of mass $w(m)$ behind, with say,

$$w(m) = 0.38 + 0.15m, \quad (10)$$

as suggested by Iben and Renzini (1983), the net contribution by the dark remnants would then be

$$\Sigma_{\text{dr}} = \int_{-\infty}^{+\infty} [\xi(\log m) - \phi_{\text{ms}}(\log m)] w(m) d \log m, \quad (11)$$

corresponding to $\Sigma_{\text{dr}} = 9 \pm 2 M_{\odot} \text{ pc}^{-2}$ for the above three IMFs (A–C). The case (C) however, produces about $11.8 M_{\odot} \text{ pc}^{-2}$ in the form of dark remnants. This may be compared with Larson's bimodal case, which gives $38 M_{\odot} \text{ pc}^{-2}$.

4.5. The returned fraction R

Given the forms of the IMF, PDMF and $w(m)$, it is easy to calculate the so-called returned fraction R for the present generation

of stars in the disc, by the formula

$$R(T_d) = \frac{\int_{-\infty}^{+\infty} [\xi(\log m) - \phi_{ms}(\log m)][m - w(m)]}{\int_{-\infty}^{+\infty} m \xi(\log m) d \log m} \quad (12)$$

Using our PDMF and IMFs and (10), we find that for case (A) with $T_d = 10$ Gyr, $R = 0.30$, and for cases (B) and (C) with $T_d = 15$ Gyr, $R = 0.426$. If the SFR is not varying a great deal, R should increase with the choice of T_d , and an approximate T_d -dependence of R may be represented by

$$R(T_d) = 0.30 + 0.025(T_d - 10 \text{ Gyr})/1 \text{ Gyr}. \quad (13)$$

Larson's bimodal IMF with Scalo's PDMF and $w(m)$ as given by (10), R turns out to be 0.65. (According to Larson's model, the total surface density of stars that have ever gone into star formation is about $173 M_\odot \text{ pc}^{-2}$, out of which the PDMF due to surviving stars only accounts for $22 M_\odot \text{ pc}^{-2}$, the dark remnants another $38 M_\odot \text{ pc}^{-2}$ leaving a total of $113 M_\odot \text{ pc}^{-2}$ to be returned to the interstellar space over the past 15 Gyr. As is obvious from (12), R is simply the ratio of 113 and 173).

4.6. Yield of metals and helium

Massive stars eject a part of their newly synthesized metals and helium to the surrounding interstellar medium, thereby enriching the latter. If a star of initial mass m throws M_{He} and m_Z amounts of helium and metals (the sum running over all elements heavier than boron) respectively into the ISM, their yields are usually defined as

$$Y_{\text{He}} = \frac{1}{1 - R} \frac{\int_{-\infty}^{+\infty} [\xi(\log m) - \phi_{ms}(\log m)] m_{\text{He}}(m) d \log m}{\int_{-\infty}^{+\infty} m \xi(\log m) d \log m}$$

and

$$Y_Z = \frac{1}{1 - R} \frac{\int_{-\infty}^{+\infty} [\xi(\log m) - \phi_{ms}(\log m)] m_Z(m) d \log m}{\int_{-\infty}^{+\infty} m \xi(\log m) d \log m}.$$

Taking the values of $m_Z(m)$ and $m_{\text{He}}(m)$ as suggested by Maeder (1984), we find that for the case (C) of our IMF, $Y_Z = 0.029$, $Y_{\text{He}} = 0.036$, the upper limit of integration in (14) chosen to be $m = 100$.

Similar calculations for Larson's bimodal IMF result in $Y_Z = 0.081$ and $Y_{\text{He}} = 0.109$. These values are considerably larger than any standard model suggestions.

The models of chemical evolution of the solar neighbourhood are found to be consistent only for $Y_Z < 0.035$ (Tinsley, 1980), otherwise it would be very difficult to reproduce the observed age-metallicity relation of Twarog (1980). Using his bimodal IMF, Larson has however computed the age-metallicity relation which seems to tally with the observed relation. Indeed, the logarithmic variation of Z with age of the disc roughly follows the right trend, but the absolute values of his Z are in fact about three times larger than the observed. Since he has normalized Z with respect to its value at the formation epoch of the solar system, the

absolute values of Z are not apparent in his diagram. It can be shown that the constant of his normalization should be about 0.06. Surely, this is too high to be regarded as the solar metallicity. The following simplified argument will explain the case in point.

We take Larson's bimodal IMF and Maeder's nucleosynthetic yield functions. On simple integration of the product of $[\xi(\log m) - \phi_{ms}(\log m)]m_Z(m)$ over $m = m_d$ and $m = 100$, it is easy to show that the total amount of metals ejected is $\Sigma_Z = 4.9 M_\odot \text{ pc}^{-2}$. Larson's IMF also results in $38 M_\odot \text{ pc}^{-2}$ for the remnants and $22 M_\odot \text{ pc}^{-2}$ for the surviving stars. Since out of a total of $173 M_\odot \text{ pc}^{-2}$ that has so far gone into stars, $113 M_\odot \text{ pc}^{-2}$ of matter had to be recycled, the actual amount of new metals that will stay back in the ISM and outside the remnants, is about $4.9(173 - 113) = 1.70 M_\odot \text{ pc}^{-2}$. This is now to be distributed among the surviving stars and the ISM, together amounting to about $29 M_\odot \text{ pc}^{-2}$ (in Larson's model). Assuming that the distribution of metals is roughly uniform, one gets a total increase in the metallicity fraction by mass in the ISM (as well as in stars) to be given by $\Delta Z \gtrsim 0.058$. Similarly for helium one gets $\Delta Y \gtrsim 0.078$. These numbers will be even larger, should we extend the upper limit of integration to somewhat above $m = 100$. Therefore, it seems that any bimodal IMF that uses too high a SFR in the past through infall for example, will have similar problem, if Maeder's values for the yields are taken. However, this overproduction of metals may be suppressed by imposing a very low upper cut off (for Larson, it has to be as low as $16 M_\odot$ only) for the function $m_Z(m)$. It should also be noted that the nucleosynthetic yields have been substantially revised (Woosley and Weaver, 1986) incorporating revised reaction rates for $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$. Using these numbers Olive et al. (1986) have recently investigated the yields for a variety of varying SFRs in the solar neighbourhood, but with no infall, in which case they observe that compared to oxygen carbon may be relatively underabundant. More investigations are needed before any firm conclusion can be drawn about effectiveness of the bimodal IMFs.

However, arguing similarly for the case (C) of our IMF, the total ejection of metal $\Sigma_Z = 1.26 M_\odot \text{ pc}^{-2}$. After partial recycling, the present amount becomes $0.60 M_\odot \text{ pc}^{-2}$. When distributed uniformly throughout $39 M_\odot \text{ pc}^{-2}$ ($= \Sigma_{\text{gas}} + \Sigma_{\text{stars}}$), this corresponds to an average rise in metallicity, $\Delta Z \gtrsim 0.0154$ over the past 15 Gyr. This fits very nicely with the idea of the initial metallicity of the disc (Rana and Wilkinson, 1986a). Since we require that the ratio of the present to the initial metallicity, that is Z_1/Z_0 , to be about 3.7 (from our G-dwarf solution without infall), it immediately follows that $Z_0 \gtrsim 0.0057$ and therefore, $Z_1 \gtrsim 0.0211$.

As for helium, $\Delta Y \gtrsim 0.019$ for the case (C) of our IMF. These enrichments in helium as well as in metals do not include the predisc enrichments, if any. Surely, there is going to be some predisc enrichments, because of the non-zero initial metallicity of the disc.

We have just made an attempt to calculate the predisc enrichments using the IMF for zero-metal stars, recently suggested by Yoshii and Saio (1986), based on the Silk's theory of opacity-limited fragmentation (Silk, 1977). Such IMFs are claimed to have a single peak around $4-10 M_\odot$, leaving therefore substantial amounts of dark remnants. Using Table 2 and 3 of Yoshii and Saio for their $\eta = 3$ case, we find that in order to develop the initial metallicity $\Delta Z = Z_0 = 0.006$, one produces $\Delta Y = Y_0 \gtrsim 0.020$, and a fraction for the dark remnants in the range of 0.02 (for their $\eta = 1.5$ model) to 0.16 (for their $\eta = 3$ model). For the sake of consistency, we took Maeder's yield functions (which also

may not be valid for Population II objects), and redid the calculation to find roughly the same value of ΔY , but a different value (0.067) for the remnant fraction. It is also possible that Population II brown dwarfs contribute substantially to the predisc dark matter (Zinnecker, 1986). A quick summary of the results of this section may be as follows:

(i) When Maeder's yields are used, Larson's bimodal IMF results in too much of metal enrichment in the solar neighbourhood, larger possibly by a factor of 3. This imposes severe constraints on the infall models or on the upper cut off for $m_z(m)$. In view of the recently revised values of the yields, more work is needed before any firm conclusion can be made.

(ii) Our IMF is consistent with no infall and requires an initial metallicity of the disc, that is again consistent with the present metallicity, and the near-constancy of the SFR.

(iii) The ratio of ΔY to ΔZ for the above zero-metal IMFs turns out to be 3.25. (It seems to be fairly independent of the η -parameter). This is quite different from the one for the Population I type of objects. For example, the above ratio for our IMF is about 1.25. It is interesting to note that a recent redetermination of this ratio from the observational study of 28 extreme metal-poor galaxies by Pagel (1985) suggests the value to be 4.5 ± 1.0 . The agreement is not bad.

(iv) The total helium correction from the big bang to the present seems to be about 0.04. This is, of course, highly tentative.

(v) During the formation of the disc, the dark remnants of the pre-disc era have possibly maintained the same proportion with the then total matter. If so, the pre-disc dark remnants have

possibly contributed a surface density of about $5 M_\odot \text{pc}^{-2}$, assuming total density of the disc at solar neighbourhood to be $75 M_\odot \text{pc}^{-2}$.

(vi) Another point may be noted. Terlevich and Melnick (1984) noted that the slope of the IMF of the massive stars in the H II regions of a sample of external galaxies becomes steeper with increasing metallicity, so that the metal poor objects show much more gradual slope than the Population I type objects. We wanted to check this point with Yoshii and Saio's IMFs for zero-metal stars. It seems that both their $\eta = 1.5$ and $\eta = 3.0$ models give much steeper slopes than -1.80 , contrary to the above expectation.

4.7. Implications for the dark matter in the solar neighbourhood

Table 4 summarises the distribution of known matter in the solar neighbourhood.

In Sect. 2, we noted that our PDMF for the main sequence stars accounted for about $32 M_\odot \text{pc}^{-2}$ with the Wielen dip and about $38 M_\odot \text{pc}^{-2}$ without it. Using Scalo's g_{ms} , the living stars in the post main sequence phase, it can be shown, contribute about $0.2-0.4 M_\odot \text{pc}^{-2}$. The interstellar medium, composed mainly of hydrogen and helium, may account for $5.7-10.8 M_\odot \text{pc}^{-2}$ (based on Table 2 of Rana and Wilkinson, 1986a), the most probable value being around $7 M_\odot \text{pc}^{-2}$. The contribution by the brown dwarfs of the disc population, as argued in Sect. 2.4, may be anything between 0 and $25 M_\odot \text{pc}^{-2}$,

Table 4. Distribution of matter in the solar neighbourhood

Type of objects/matter	Contribution to the local surface density in $M_\odot \text{pc}^{-2}$	
	Estimated ranges	Preferred for this work
1. PDMF of main sequence stars of Population I ($0.08 \leq m \leq 100$): present work with/without Wielen dip	$\frac{32}{38}$ (13–39)	35 ± 3 (22)
2. Post-main sequence stars of Population I	0.2–0.4	0.3 ± 0.1
3. Brown dwarfs	0–25	$\gtrsim 7.5(?)$
4. Gas($\text{H I} + \text{H}_2 + \text{He}$) + dust	5.7–10.8	7.0 ± 0.8
5. Dark remnants of Population I: present work Larson's bimodal IMF	6.8–11.8 (38)	9 ± 2
6. Metal-poor stars	2–6	4 ± 1
7. Dark remnants of predisc generation (from the requirement of initial metallicity of the disc)	1.5–10.5	5 ± 3
Total (accountable)	50–100	68 ± 10
Total required	60–150	70 ± 5

but the recent works of Hawkins and Gilmore et al., coupled with those of Boss and van de Kamp suggest that the amount may be $> 7.5 M_{\odot} \text{pc}^{-2}$. The surviving metal-poor stars that co-exist with the disc may contribute another $2 \pm 1 M_{\odot} \text{pc}^{-2}$ (taking numbers from Vader and de Jong, 1981 and Bahcall, 1986). The dark remnants of the disc population, mostly dead white dwarfs, could give anything in the range of $5\text{--}10 M_{\odot} \text{pc}^{-2}$. The three possible forms of the IMFs that are noted in this work allow the range to be $6.8\text{--}11.8 M_{\odot} \text{pc}^{-2}$, although Larson's bimodal IMF can generate any amount up to about $40 M_{\odot} \text{pc}^{-2}$. From the requirement of our initial metallicity, the predisc generation might have left some dark remnants, a rough estimate of which suggests $1.5\text{--}10.5 M_{\odot} \text{pc}^{-2}$. It is argued in the previous subsection that a likely number may be $5 \pm 3 M_{\odot} \text{pc}^{-2}$. Therefore, the above distribution does account for about $68 M_{\odot} \text{pc}^{-2}$, with the actual figure most probably lying in the range $50\text{--}100 M_{\odot} \text{pc}^{-2}$. To compare with the dynamical requirements, the total surface density should be anything in the range $60\text{--}150 M_{\odot} \text{pc}^{-2}$, with the most probable value, $70 \pm 5 M_{\odot} \text{pc}^{-2}$ (Bahcall et al., 1983). So the required and the observed amounts seem to agree reasonably well.

5. Conclusions

The present work aimed to derive the mass functions of the main sequence stars in the solar neighbourhood, adopting a different set of new input data not included in Scalo (1986)'s latest review. The particular findings are as follows:

- (i) The luminosity function seems to have been somewhat underestimated by Scalo (1986), particularly for the massive stars.
- (ii) The contribution to the total surface density of matter in the solar neighbourhood by stars of Population I seems to be about $32 M_{\odot} \text{pc}^{-2}$, compared to Scalo's $22 M_{\odot} \text{pc}^{-2}$. The difference is primarily due to the change in scale heights between the two works.
- (iii) The contribution by the brown dwarfs still remains an open issue, although out of a possible range of $0\text{--}25 M_{\odot} \text{pc}^{-2}$, the actual contribution may be $\gtrsim 7.5 M_{\odot} \text{pc}^{-2}$. If so, the brown dwarf number density would be about one (of mass $\sim 0.05 M_{\odot}$) every 8pc^3 or so. If these brown dwarfs are not of the Population II origin, Larson's model of cloud fragmentation may not be viable.
- (iv) The initial mass function may have an overall slope of -1.80 over the entire range of $1.5\text{--}100 M_{\odot}$.
- (v) The IMF possibly shows a few small humps, suggesting that it could be multimodal.
- (vi) Larson's bimodal IMF is compared with ours to find that both the returned fraction R and the yield of metals Y_Z are substantially larger for the former than would be required by the simple models of the chemical evolution of the solar neighbourhood.
- (vii) The present work seems to account for the total dynamical requirement for the total surface density of matter in the solar neighbourhood in the form of stellar populations, dead or alive, and interstellar gas only.

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